CHAPTER – 27
SPECIFIC HEAT CAPACITIES OF GASES

1. N = 1 mole,   W = 20 g/mol,   V = 50 m/s

   K.E. of the vessel = Internal energy of the gas

   $= (1/2) mv^2 = (1/2) \times 20 \times 10^{-3} \times 50 \times 50 = 25$ J

   $25 = n\dfrac{3}{2}r(\Delta T) \Rightarrow 25 = 1 \times \dfrac{3}{2} \times 8.31 \times \Delta T \Rightarrow \Delta T = \dfrac{50}{3 \times 8.3} \approx 2$ k.

2. m = 5 g, $\Delta t = 25 - 15 = 10°C$

   $C_V = 0.172$ cal/g-°CJ = 4.2 J/Cal.

   dQ = du + dw

   Now, V = 0 (for a rigid body)

   So, dw = 0.

   So dQ = du.

   Q = msdt = 5 × 0.172 × 10 = 8.6 cal = 8.6 × 4.2 = 36.12 Joule.

3. $\gamma = 1.4$,   w or piston = 50 kg.,   A of piston = 100 cm^2

   Po = 100 kpa,   g = 10 m/s^2,   x = 20 cm.

   $dw = pdv = \left(\dfrac{mg}{A} + Po\right)Adx = \left(\dfrac{50 \times 10}{100 \times 10^{-4}} + 10^5\right)100 \times 10^{-4} \times 20 \times 10^{-2} = 1.5 \times 10^5 \times 20 \times 10^{-4} = 300$ J.

   $nRdt = 300 \Rightarrow dT = \dfrac{300}{nR}$

   $dQ = nCpdT = nCp \times \dfrac{300}{nR} = \dfrac{n\gamma R300}{(\gamma - 1)nR} = \dfrac{300 \times 1.4}{0.4} = 1050$ J.

4. $C_V H_2 = 2.4$ Cal/g°C, $C_P H^2 = 3.4$ Cal/g°C

   M = 2 g/ Mol,   R = 8.3 × 10^7 erg/mol-°C

   We know, $C_P - C_V = 1$ Cal/g°C

   So, difference of molar specific heats

   $= C_P \times M - C_V \times M = 1 \times 2 = 2$ Cal/g°C

   Now, $2 \times J = R \Rightarrow 2 \times J = 8.3 \times 10^7$ erg/mol-°C $\Rightarrow J = 4.15 \times 10^7$ erg/cal.

5. $\dfrac{C_P}{C_V} = 7.6$, n = 1 mole, $\Delta T = 50K$

   (a) Keeping the pressure constant, dQ = du + dw,

        $\Delta T = 50$ K, $\gamma = 7/6$,   m = 1 mole,

        $dQ = du + dw \Rightarrow nC_VdT = du + RdT \Rightarrow du = nCpdT - RdT$

        $= 1 \times \dfrac{R\gamma}{\gamma - 1} \times dT - RdT = \dfrac{R \times \dfrac{7}{6}}{\dfrac{7}{6} - 1}dT - RdT$

        $= DT - RdT = 7RdT - RdT = 6 RdT = 6 \times 8.3 \times 50 = 2490$ J.

   (b) Kipping Volume constant, $dv = nC_VdT$

        $= 1 \times \dfrac{R}{\gamma - 1} \times dt = \dfrac{1 \times 8.3}{\dfrac{7}{6} - 1} \times 50$

        $= 8.3 \times 50 \times 6 = 2490$ J

   (c) Adiabetically dQ = 0,   du = – dw

        $= \left[\dfrac{n \times R}{\gamma - 1}(T_1 - T_2)\right] = \dfrac{1 \times 8.3}{\dfrac{7}{6} - 1}(T_2 - T_1) = 8.3 \times 50 \times 6 = 2490$ J

6. m = 1.18 g, $\qquad$ V = 1 × 10^3 cm^3 = 1 L $\quad$ T = 300 k, $\qquad$ P = 10^5 Pa

    PV = nRT $\quad$ or $\qquad$ n = $\dfrac{PV}{RT}$ = 10^5 = atm.

    N = $\dfrac{PV}{RT}$ = $\dfrac{1}{8.2 \times 10^{-2} \times 3 \times 10^2}$ = $\dfrac{1}{8.2 \times 3}$ = $\dfrac{1}{24.6}$

    Now, $C_v = \dfrac{1}{n} \times \dfrac{Q}{dt}$ = 24.6 × 2 = 49.2

    C_p = R + C_v = 1.987 + 49.2 = 51.187

    Q = $nC_p dT$ = $\dfrac{1}{24.6} \times 51.187 \times 1$ = 2.08 Cal.

7. V_1 = 100 cm^3, $\qquad$ V_2 = 200 cm^3 $\quad$ P = 2 × 10^5 Pa, ΔQ = 50J

    (a) ΔQ = du + dw $\Rightarrow$ 50 = du + 20× 10^5(200 – 100 × 10^{-6}) $\Rightarrow$ 50 = du + 20 $\Rightarrow$ du = 30 J

    (b) 30 = n × $\dfrac{3}{2}$ × 8.3 × 300 $\quad$ [U = $\dfrac{3}{2}$nRT for monoatomic]

    $\Rightarrow$ n = $\dfrac{2}{3 \times 83}$ = $\dfrac{2}{249}$ = 0.008

    (c) du = $nC_v dT \Rightarrow C_v$ = $\dfrac{dndTu}{}$ = $\dfrac{30}{0.008 \times 300}$ = 12.5

    C_p = C_v + R = 12.5 + 8.3 = 20.3

    (d) C_v = 12.5 (Proved above)

8. Q = Amt of heat given

    Work done = $\dfrac{Q}{2}$, $\quad$ ΔQ = W + Δ U

    for monoatomic gas $\Rightarrow \Delta U$ = Q – $\dfrac{Q}{2}$ = $\dfrac{Q}{2}$

    V = $n\dfrac{3}{2}RT$ = $\dfrac{Q}{2}$ = nT× $\dfrac{3}{2}$R = 3R × nT

    Again Q = n CpdT Where C_P > Molar heat capacity at const. pressure.

    3RnT = ndT$C_P \Rightarrow C_P$ = 3R

9. P = KV $\Rightarrow \dfrac{nRT}{V}$ = KV $\Rightarrow$ RT = KV^2 $\Rightarrow$ R ΔT = 2KV ΔU $\Rightarrow \dfrac{R\Delta T}{2KV}$ = dv

    dQ = du + dw $\Rightarrow$ mcdT = C_vdT + pdv $\Rightarrow$ msdT = C_V dT+ $\dfrac{PRdF}{2KV}$

    $\Rightarrow$ ms = C_V + $\dfrac{RKV}{2KV} \Rightarrow C_P$ + $\dfrac{R}{2}$

10. $\dfrac{C_P}{C_V}$ = γ, $\quad$ C_P – C_V = R, $\qquad$ C_V = $\dfrac{r}{\gamma - 1}$, $\qquad$ C_P = $\dfrac{\gamma R}{\gamma - 1}$

    Pdv = $\dfrac{1}{b+1}\left(Rdt\right)$

    $\Rightarrow$ 0 = C_VdT + $\dfrac{1}{b+1}\left(Rdt\right) \Rightarrow \dfrac{1}{b+1}$ = $\dfrac{-C_V}{R}$

    $\Rightarrow$ b + 1 = $\dfrac{-R}{C_V}$ = $\dfrac{-(C_P - C_V)}{C_V}$ = $-\gamma$ +1 $\Rightarrow$ b = $-\gamma$

11. Considering two gases, in Gas(1) we have,

    γ, Cp$_1$ (Sp. Heat at const. 'P'), Cv$_1$ (Sp. Heat at const. 'V'), n$_1$ (No. of moles)

    $\dfrac{Cp_1}{Cv_1} = \gamma$ & Cp$_1$ – Cv$_1$ = R

$\Rightarrow \gamma Cv_1 - Cv_1 = R \Rightarrow Cv_1 (\gamma - 1) = R$

$\Rightarrow Cv_1 = \dfrac{R}{\gamma - 1}$ & $Cp_1 = \dfrac{\gamma R}{\gamma - 1}$

In Gas(2) we have, γ, Cp_2 (Sp. Heat at const. 'P'), Cv_2 (Sp. Heat at const. 'V'), n_2 (No. of moles)

$\dfrac{Cp_2}{Cv_2} = \gamma$ & $Cp_2 - Cv_2 = R \Rightarrow \gamma Cv_2 - Cv_2 = R \Rightarrow Cv_2 (\gamma - 1) = R \Rightarrow Cv_2 = \dfrac{R}{\gamma - 1}$ & $Cp_2 = \dfrac{\gamma R}{\gamma - 1}$

Given $n_1 : n_2 = 1 : 2$

$dU_1 = nCv_1 dT$ & $dU_2 = 2nCv_2 dT = 3nCvdT$

$\Rightarrow C_V = \dfrac{Cv_1 + 2Cv_2}{3} = \dfrac{\dfrac{R}{\gamma - 1} + \dfrac{2R}{\gamma - 1}}{3} = \dfrac{3R}{3(\gamma - 1)} = \dfrac{R}{\gamma - 1}$ \qquad ...(1)

& $Cp = \gamma Cv = \dfrac{\gamma r}{\gamma - 1}$ ...(2)

So, $\dfrac{Cp}{Cv} = \gamma$ [from (1) & (2)]

12. $Cp' = 2.5 R Cp'' = 3.5 R$

$Cv' = 1.5 R \qquad Cv'' = 2.5 R$

$n_1 = n_2 = 1$ mol $\qquad (n_1 + n_2)C_V dT = n_1 C_v dT + n_2 C_v dT$

$\Rightarrow C_V = \dfrac{n_1 Cv' + n_2 Cv''}{n_1 + n_2} = \dfrac{1.5R + 2.5R}{2} = 2R$

$C_P = C_V + R = 2R + R = 3R$

$\gamma = \dfrac{C_p}{C_V} = \dfrac{3R}{2R} = 1.5$

13. $n = \dfrac{1}{2}$ mole, $\qquad R = \dfrac{25}{3}$ J/mol-k, $\qquad \gamma = \dfrac{5}{3}$

(a) Temp at A = T_a, $P_a V_a = nRT_a$

$\Rightarrow T_a = \dfrac{P_a V_a}{nR} = \dfrac{5000 \times 10^{-6} \times 100 \times 10^3}{\dfrac{1}{2} \times \dfrac{25}{3}} = 120$ k.

Similarly temperatures at point b = 240 k at C it is 480 k and at D it is 240 k.

(b) For ab process,

$dQ = nCpdT$ \qquad [since ab is isobaric]

$= \dfrac{1}{2} \times \dfrac{R\gamma}{\gamma - 1}(T_b - T_a) = \dfrac{1}{2} \times \dfrac{\dfrac{35}{3} \times \dfrac{5}{3}}{\dfrac{5}{3} - 1} \times (240 - 120) = \dfrac{1}{2} \times \dfrac{125}{9} \times \dfrac{3}{2} \times 120 = 1250$ J

For bc, $\qquad dQ = du + dw$ \quad [dq = 0, Isochorie process]

$\Rightarrow dQ = du = nC_v dT = \dfrac{nR}{\gamma - 1}(T_c - T_a) = \dfrac{1}{2} \times \dfrac{\dfrac{25}{3}}{\left(\dfrac{5}{3} - 1\right)}(240) = \dfrac{1}{2} \times \dfrac{25}{3} \times \dfrac{3}{2} \times 240 = 1500$ J

(c) Heat liberated in cd = $- nC_p dT$

$= \dfrac{-1}{2} \times \dfrac{nR}{\gamma - 1}(T_d - T_c) = \dfrac{-1}{2} \times \dfrac{125}{3} \times \dfrac{3}{2} \times 240 = 2500$ J

Heat liberated in da = $- nC_v dT$

$= \dfrac{-1}{2} \times \dfrac{R}{\gamma - 1}(T_a - T_d) = \dfrac{-1}{2} \times \dfrac{25}{2} \times (120 - 240) = 750$ J
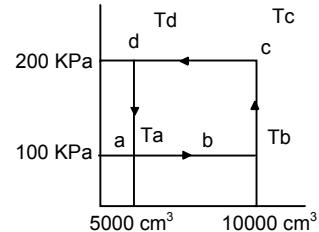
14. (a) For a, b 'V' is constant

So, $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2} \Rightarrow \dfrac{100}{300} = \dfrac{200}{T_2} \Rightarrow T_2 = \dfrac{200 \times 300}{100} = 600\ k$

For b,c 'P' is constant

So, $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2} \Rightarrow \dfrac{100}{600} = \dfrac{150}{T_2} \Rightarrow T_2 = \dfrac{600 \times 150}{100} = 900\ k$
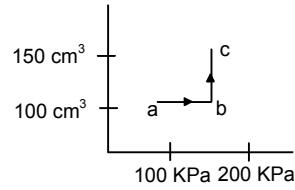
(b) Work done = Area enclosed under the graph 50 cc × 200 kpa = $50 \times 10^{-6} \times 200 \times 10^3\ J = 10\ J$

(c) 'Q' Supplied = $nC_v dT$

Now, $n = \dfrac{PV}{RT}$ considering at pt. 'b'

$C_v = \dfrac{R}{\gamma - 1} dT = 300\ a,\ b.$

$Q_{bc} = \dfrac{PV}{RT} \times \dfrac{R}{\gamma - 1} dT = \dfrac{200 \times 10^3 \times 100 \times 10^{-6}}{600 \times 0.67} \times 300 = 14.925$ $(\because \gamma = 1.67)$

Q supplied to be $nC_p dT$ $[\because C_p = \dfrac{\gamma R}{\gamma - 1}]$

$= \dfrac{PV}{RT} \times \dfrac{\gamma R}{\gamma - 1} dT = \dfrac{200 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 900} \times \dfrac{1.67 \times 8.3}{0.67} \times 300 = 24.925$

(d) $Q = \Delta U + w$

Now, $\Delta U = Q - w$ = Heat supplied – Work done = (24.925 + 14.925) – 1 = 29.850

15. In Joly's differential steam calorimeter

$C_v = \dfrac{m_2 L}{m_1 (\theta_2 - \theta_1)}$

m_2 = Mass of steam condensed = 0.095 g, L = 540 Cal/g = 540 × 4.2 J/g

m_1 = Mass of gas present = 3 g, $\theta_1 = 20°C$, $\theta_2 = 100°C$

$\Rightarrow C_v = \dfrac{0.095 \times 540 \times 4.2}{3(100 - 20)} = 0.89 \approx 0.9$ J/g-K

16. $\gamma = 1.5$

Since it is an adiabatic process, So PV^γ = const.

(a) $P_1 V_1^\gamma = P_2 V_2^\gamma$ Given V_1 = 4 L, V_2 = 3 L, $\dfrac{P_2}{P_1} = ?$

$\Rightarrow \dfrac{P_2}{P_1} = \left(\dfrac{V_1}{V_2}\right)^\gamma = \left(\dfrac{4}{3}\right)^{1.5} = 1.5396 \approx 1.54$

(b) $TV^{\gamma-1}$ = Const.

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow \dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = \left(\dfrac{4}{3}\right)^{0.5} = 1.154$

17. $P_1 = 2.5 \times 10^5$ Pa, V_1 = 100 cc, T_1 = 300 k

(a) $P_1 V_1^\gamma = P_2 V_2^\gamma$

$\Rightarrow 2.5 \times 10^5 \times V^{1.5} = \left(\dfrac{V}{2}\right)^{1.5} \times P_2$

$\Rightarrow P_2 = 2^{1.5} \times 2.5 \times 10^5 = 7.07 \times 10^5 \approx 7.1 \times 10^5$

(b) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow 300 \times (100)^{1.5-1} = T_2 \times (50)^{1.5-1}$

$\Rightarrow T_2 = \dfrac{3000}{7.07} = 424.32\ k \approx 424\ k$

(c) Work done by the gas in the process

$$W = \frac{mR}{\gamma - 1}[T_2 - T_1] = \frac{P_1 V_1}{T(\gamma - 1)}[T_2 - T_1]$$

$$= \frac{2.5 \times 10^5 \times 100 \times 10^{-6}}{300(1.5 - 1)}[424 - 300] = \frac{2.5 \times 10}{300 \times 0.5} \times 124 = 20.72 \approx 21 \text{ J}$$

18. $\gamma = 1.4$, $\quad T_1 = 20°C = 293$ k, $\quad P_1 = 2$ atm, $\quad p_2 = 1$ atm

We know for adiabatic process,

$P_1^{1-\gamma} \times T_1^{\gamma} = P_2^{1-\gamma} \times T_2^{\gamma}$ or $(2)^{1-1.4} \times (293)^{1.4} = (1)^{1-1.4} \times T_2^{1.4}$

$\Rightarrow (2)^{0.4} \times (293)^{1.4} = T_2^{1.4} \Rightarrow 2153.78 = T_2^{1.4} \Rightarrow T_2 = (2153.78)^{1/1.4} = 240.3$ K

19. $P_1 = 100$ KPa $= 10^5$ Pa, $\quad V_1 = 400 \text{ cm}^3 = 400 \times 10^{-6} \text{ m}^3$, $\quad T_1 = 300$ k,

$$\gamma = \frac{C_P}{C_V} = 1.5$$

(a) Suddenly compressed to $V_2 = 100 \text{ cm}^3$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \quad \Rightarrow 10^5 (400)^{1.5} = P_2 \times (100)^{1.5}$

$\Rightarrow P_2 = 10^5 \times (4)^{1.5} = 800$ KPa

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow 300 \times (400)^{1.5-1} = T_2 \times (100)^{1.5-1} \Rightarrow T_2 = \frac{300 \times 20}{10} = 600$ K

(b) Even if the container is slowly compressed the walls are adiabatic so heat transferred is 0.
Thus the values remain, $P_2 = 800$ KPa, $\quad T_2 = 600$ K.

20. Given $\dfrac{C_P}{C_V} = \gamma \qquad P_0$ (Initial Pressure), $\qquad V_0$ (Initial Volume)

(a) (i) Isothermal compression, $P_1 V_1 = P_2 V_2$ or, $P_0 V_0 = \dfrac{P_2 V_0}{2} \Rightarrow P_2 = 2P_0$

(ii) Adiabatic Compression $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$ or $2P_0 \left(\dfrac{V_0}{2}\right)^{\gamma} = P'\left(\dfrac{V_0}{4}\right)^{\gamma}$

$\Rightarrow P' = \dfrac{V_0^{\gamma}}{2^{\gamma}} \times 2P_0 \times \dfrac{4^{\gamma}}{V_0^{\gamma}} = 2^{\gamma} \times 2 P_0 \Rightarrow P_0 2^{\gamma+1}$

(b) (i) Adiabatic compression $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$ or $P_0 V_0^{\gamma} = P'\left(\dfrac{V_0}{2}\right)^{\gamma} \Rightarrow P' = P_0 2^{\gamma}$

(ii) Isothermal compression $P_1 V_1 = P_2 V_2$ or $2^{\gamma} P_0 \times \dfrac{V_0}{2} = P_2 \times \dfrac{V_0}{4} \Rightarrow P_2 = P_0 2^{\gamma+1}$

21. Initial pressure $= P_0$

Initial Volume $= V_0$

$$\gamma = \frac{C_P}{C_V}$$

(a) Isothermally to pressure $\dfrac{P_0}{2}$

$P_0 V_0 = \dfrac{P_0}{2} V_1 \Rightarrow V_1 = 2 V_0$

Adiabetically to pressure $= \dfrac{P_0}{4}$

$\dfrac{P_0}{2}(V_1)^{\gamma} = \dfrac{P_0}{4}(V_2)^{\gamma} \Rightarrow \dfrac{P_0}{2}(2V_0)^{\gamma} = \dfrac{P_0}{4}(V_2)^{\gamma}$

$\Rightarrow 2^{\gamma+1} V_0^{\gamma} = V_2^{\gamma} \Rightarrow V_2 = 2^{(\gamma+1)/\gamma} V_0$

$\therefore$ Final Volume $= 2^{(\gamma+1)/\gamma} V_0$

(b) Adiabetically to pressure $\dfrac{P_0}{2}$ to P_0

$$P_0 \times (2^{\gamma+1} V_0{}^{\gamma}) = \dfrac{P_0}{2} \times (V')^{\gamma}$$

Isothermal to pressure $\dfrac{P_0}{4}$

$$\dfrac{P_0}{2} \times 2^{1/\gamma} V_0 = \dfrac{P_0}{4} \times V'' \Rightarrow V'' = 2^{(\gamma+1)/\gamma} V_0$$

$\therefore$ Final Volume $= 2^{(\gamma+1)/\gamma} V_0$

22. $PV = nRT$

Given $P = 150 \text{ KPa} = 150 \times 10^3 \text{ Pa}$,　　　$V = 150 \text{ cm}^3 = 150 \times 10^{-6} \text{ m}^3$,　　$T = 300 \text{ k}$

(a) $n = \dfrac{PV}{RT} = \dfrac{150 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 300} = 9.036 \times 10^{-3} = 0.009$ moles.

(b) $\dfrac{C_P}{C_V} = \gamma \Rightarrow \dfrac{\gamma R}{(\gamma-1)C_V} = \gamma$　　　$\left[\therefore C_P = \dfrac{\gamma R}{\gamma - 1} \right]$

$\Rightarrow C_V = \dfrac{R}{\gamma - 1} = \dfrac{8.3}{1.5 - 1} = \dfrac{8.3}{0.5} = 2R = 16.6 \text{ J/mole}$

(c) Given $P_1 = 150 \text{ KPa} = 150 \times 10^3 \text{ Pa}$,　　$P_2 = ?$

$V_1 = 150 \text{ cm}^3 = 150 \times 10^{-6} \text{ m}^3$,　　$\gamma = 1.5$

$V_2 = 50 \text{ cm}^3 = 50 \times 10^{-6} \text{ m}^3$,　　$T_1 = 300 \text{ k}$,　　$T_2 = ?$

Since the process is adiabatic Hence $- P_1 V_1{}^{\gamma} = P_2 V_2{}^{\gamma}$

$\Rightarrow 150 \times 10^3 (150 \times 10^{-6})^{\gamma} = P_2 \times (50 \times 10^{-6})^{\gamma}$

$\Rightarrow P_2 = 150 \times 10^3 \times \left(\dfrac{150 \times 10^{-6}}{50 \times 10^{-6}} \right)^{1.5} = 150000 \times 3^{1.5} = 779.422 \times 10^3 \text{ Pa} \approx 780 \text{ KPa}$

(d) $\Delta Q = W + \Delta U$ or $W = -\Delta U$　　　$[\therefore \Delta U = 0$, in adiabatic$]$

$= - nC_V dT = - 0.009 \times 16.6 \times (520 - 300) = - 0.009 \times 16.6 \times 220 = - 32.8 \text{ J} \approx - 33 \text{ J}$

(e) $\Delta U = nC_V dT = 0.009 \times 16.6 \times 220 \approx 33 \text{ J}$

23. $V_A = V_B = V_C$

For A, the process is isothermal

$$P_A V_A = P_A' V_A' \Rightarrow P_A' = P_A \dfrac{V_A}{V_A{}'} = P_A \times \dfrac{1}{2}$$

For B, the process is adiabatic,

$$P_A (V_B)^{\gamma} = P_A'(V_B)^{\gamma} = P_B' = P_B \left(\dfrac{V_B}{V_B{}'} \right)^{\gamma} = P_B \times \left(\dfrac{1}{2} \right)^{1.5} = \dfrac{P_B}{2^{1.5}}$$

For, C, the process is isobaric

$$\dfrac{V_C}{T_C} = \dfrac{V_C{}'}{T_C{}'} \Rightarrow \dfrac{V_C}{T_C} = \dfrac{2V_C{}'}{T_C{}'} \Rightarrow T_C' = \dfrac{2}{T_C}$$

Final pressures are equal.

$= \dfrac{p_A}{2} = \dfrac{P_B}{2^{1.5}} = P_C \Rightarrow P_A : P_B : P_C = 2 : 2^{1.5} : 1 = 2 : 2\sqrt{2} : 1$

24. P_1 = Initial Pressure　　　V_1 = Initial Volume　　　P_2 = Final Pressure　　　V_2 = Final Volume

Given, $V_2 = 2V_1$, Isothermal workdone $= nRT_1 \text{Ln} \left(\dfrac{V_2}{V_1} \right)$

Adiabatic workdone = $\dfrac{P_1V_1 - P_2V_2}{\gamma - 1}$

Given that workdone in both cases is same.

Hence $nRT_1 \, Ln\left(\dfrac{V_2}{V_1}\right) = \dfrac{P_1V_1 - P_2V_2}{\gamma - 1} \Rightarrow (\gamma - 1) \ln\left(\dfrac{V_2}{V_1}\right) = \dfrac{P_1V_1 - P_2V_2}{nRT_1}$

$\Rightarrow (\gamma - 1)\ln\left(\dfrac{V_2}{V_1}\right) = \dfrac{nRT_1 - nRT_2}{nRT_1} \Rightarrow (\gamma - 1) \ln 2 = \dfrac{T_1 - T_1}{T_1}$ …(i) [$\because V_2 = 2V_1$]

We know $TV^{\gamma - 1}$ = const. in adiabatic Process.

$T_1V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$, or $T_1 (V_2)^{\gamma - 1} = T_2 \times (2)^{\gamma - 1} \times (V_1)^{\gamma - 1}$

Or, $T_1 = 2^{\gamma - 1} \times T_2$ or $T_2 = T_1^{1 - \gamma}$ …(ii)

From (i) & (ii)

$(\gamma - 1) \ln 2 = \dfrac{T_1 - T_1 \times 2^{1 - \gamma}}{T_1} \Rightarrow (\gamma - 1) \ln 2 = 1 - 2^{1 - \gamma}$

25. $\gamma = 1.5$, T = 300 k, $V = 1 L v = \dfrac{1}{2} l$

(a) The process is adiabatic as it is sudden,

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow P_1 (V_0)^{\gamma} = P_2 \left(\dfrac{V_0}{2}\right)^{\gamma} \Rightarrow P_2 = P_1 \left(\dfrac{1}{1/2}\right)^{1.5} = P_1 (2)^{1.5} \Rightarrow \dfrac{P_2}{P_1} = 2^{1.5} = 2\sqrt{2}$

(b) $P_1 = 100$ KPa $= 10^5$ Pa $W = \dfrac{nR}{\gamma - 1}[T_1 - T_2]$

$T_1 V_1^{\gamma - 1} = P_2 V_2^{\gamma - 1} \Rightarrow 300 \times (1)^{1.5 - 1} = T_2 (0.5)^{1.5 - 1} \Rightarrow 300 \times 1 = T_2 \sqrt{0.5}$

$T_2 = 300 \times \sqrt{\dfrac{1}{0.5}} = 300\sqrt{2}$ K

$P_1 V_1 = nRT_1 \Rightarrow n = \dfrac{P_1V_1}{RT_1} = \dfrac{10^5 \times 10^{-3}}{R \times 300} = \dfrac{1}{3R}$ (V in m³)

$w = \dfrac{nR}{\gamma - 1}[T_1 - T_2] = \dfrac{1R}{3R(1.5 - 1)}\left[300 - 300\sqrt{2}\right] = \dfrac{300}{3 \times 0.5}\left(1 - \sqrt{2}\right) = -82.8 \text{ J} \approx -82 \text{ J}.$

(c) Internal Energy,

$dQ = 0$, $\Rightarrow du = -dw = -(-82.8)J = 82.8 \text{ J} \approx 82 \text{ J}.$

(d) Final Temp $= 300\sqrt{2} = 300 \times 1.414 \times 100 = 424.2 \text{ k} \approx 424 \text{ k}.$

(e) The pressure is kept constant. $\therefore$ The process is isobaric.

Work done $= nRdT = \dfrac{1}{3R} \times R \times (300 - 300\sqrt{2})$ Final Temp = 300 K

$= -\dfrac{1}{3} \times 300 (0.414) = -41.4 \text{ J}.$ Initial Temp $= 300\sqrt{2}$

(f) Initial volume $\Rightarrow \dfrac{V_1}{T_1} = \dfrac{V_1'}{T_1'} = V_1' = \dfrac{V_1}{T_1} \times T_1' = \dfrac{1}{2 \times 300 \times \sqrt{2}} \times 300 = \dfrac{1}{2\sqrt{2}}$ L.

Final volume = 1L

Work done in isothermal $= nRT\ln\dfrac{V_2}{V_1}$

$= \dfrac{1}{3R} \times R \times 300 \ln\left(\dfrac{1}{1/2\sqrt{2}}\right) = 100 \times \ln\left(2\sqrt{2}\right) = 100 \times 1.039 \approx 103$

(g) Net work done $= W_A + W_B + W_C = -82 - 41.4 + 103 = -20.4$ J.

26. Given $\gamma = 1.5$

   We know fro adiabatic process $TV^{\gamma-1}$ = Const.

    So, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ ...(eq)

    As, it is an adiabatic process and all the other conditions are same. Hence the above equation can be applied.

    So, $T_1 \times \left(\dfrac{3V}{4}\right)^{1.5-1} = T_2 \times \left(\dfrac{V}{4}\right)^{1.5-1} \Rightarrow T_1 \times \left(\dfrac{3V}{4}\right)^{0.5} = T_2 \times \left(\dfrac{V}{4}\right)^{0.5}$

    $\Rightarrow \dfrac{T_1}{T_2} = \left(\dfrac{V}{4}\right)^{0.5} \times \left(\dfrac{4}{3V}\right)^{0.5} = \dfrac{1}{\sqrt{3}}$ So, $T_1 : T_2 = 1 : \sqrt{3}$

27. $V = 200$ cm^3, C = 12.5 J/mol-k, T = 300 k, P = 75 cm

    (a) No. of moles of gas in each vessel,

    $\dfrac{PV}{RT} = \dfrac{75 \times 13.6 \times 980 \times 200}{8.3 \times 10^7 \times 300} = 0.008$

    (b) Heat is supplied to the gas but $dv = 0$

    $dQ = du \Rightarrow 5 = nC_V dT \Rightarrow 5 = 0.008 \times 12.5 \times dT \Rightarrow dT = \dfrac{5}{0.008 \times 12.5}$ for (A)

    For (B) $dT = \dfrac{10}{0.008 \times 12.5}$ $\therefore \dfrac{P}{T} = \dfrac{P_A}{T_A}$ [For container A]

    $\Rightarrow \dfrac{75}{300} = \dfrac{P_A \times 0.008 \times 12.5}{5} \Rightarrow P_A = \dfrac{75 \times 5}{300 \times 0.008 \times 12.5} = 12.5$ cm of Hg.

    $\therefore \dfrac{P}{T} = \dfrac{P_B}{T_B}$ [For Container B] $\Rightarrow \dfrac{75}{300} = \dfrac{P_B \times 0.008 \times 12.5}{10} \Rightarrow P_B = 2 P_A = 25$ cm of Hg.

    Mercury moves by a distance $P_B - P_A = 25 - 12.5 = 12.5$ Cm.

28. mHe = 0.1 g, $\gamma = 1.67$, $\mu = 4$ g/mol, mH$_2$ =?

    $\mu = 28$/mol $\gamma_2 = 1.4$

    Since it is an adiabatic surrounding

    He $dQ = nC_V dT = \dfrac{0.1}{4} \times \dfrac{R}{\gamma-1} \times dT = \dfrac{0.1}{4} \times \dfrac{R}{(1.67-1)} \times dT$ ...(i)

    $H_2 = nC_V dT = \dfrac{m}{2} \times \dfrac{R}{\gamma-1} \times dT = \dfrac{m}{2} \times \dfrac{R}{1.4-1} \times dT$ [Where m is the rqd.

    Mass of H$_2$]

    Since equal amount of heat is given to both and ΔT is same in both.

    Equating (i) & (ii) we get

    $\dfrac{0.1}{4} \times \dfrac{R}{0.67} \times dT = \dfrac{m}{2} \times \dfrac{R}{0.4} \times dT \Rightarrow m = \dfrac{0.1}{2} \times \dfrac{0.4}{0.67} = 0.0298 \approx 0.03$ g

29. Initial pressure = P_0, Initial Temperature = T_0

    Initial Volume = V_0

    $\dfrac{C_P}{C_V} = \gamma$

    (a) For the diathermic vessel the temperature inside remains constant

    $P_1 V_1 - P_2 V_2 \Rightarrow P_0 V_0 = P_2 \times 2V_0 \Rightarrow P_2 = \dfrac{P_0}{2}$, Temperature = T_o

    For adiabatic vessel the temperature does not remains constant. The process is adiabatic

    $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_0 V_0^{\gamma-1} = T_2 \times (2V_0)^{\gamma-1} \Rightarrow T_2 = T_0 \left(\dfrac{V_0}{2V_0}\right)^{\gamma-1} = T_0 \times \left(\dfrac{1}{2}\right)^{\gamma-1} = \dfrac{T_0}{2^{\gamma-1}}$
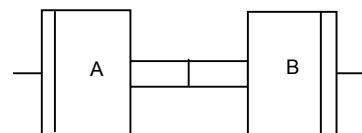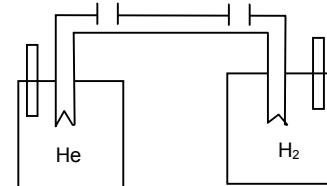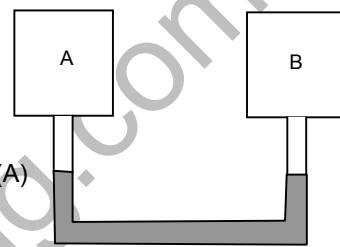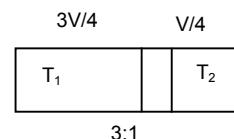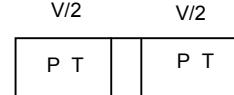
$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow P_0 V_0^{\gamma} = p_1 (2V_0)^{\gamma} \Rightarrow P_1 = P_0 \left(\dfrac{V_0}{2V_0}\right)^{\gamma} = \dfrac{P_0}{2^{\gamma}}$

(b) When the values are opened, the temperature remains T_0 through out

$P_1 = \dfrac{n_1 RT_0}{4V_0}$, $P_2 = \dfrac{n_2 RT_0}{4V_0}$ [Total value after the expt = $2V_0 + 2V_0 = 4V_0$]

$P = P_1 + P_2 = \dfrac{(n_1 + n_2)RT_0}{4V_0} = \dfrac{2nRT_0}{4V_0} = \dfrac{nRT_0}{2V} = \dfrac{P_0}{2}$

30. For an adiabatic process, Pv^{γ} = Const.

There will be a common pressure 'P' when the equilibrium is reached

Hence $P_1 \left(\dfrac{V_0}{2}\right)^{\gamma} = P(V')^{\gamma}$

For left $P = P_1 \left(\dfrac{V_0}{2}\right)^{\gamma} (V')^{\gamma}$ …(1)

For Right $P = P_2 \left(\dfrac{V_0}{2}\right)^{\gamma} (V_0 - V')^{\gamma}$ …(2)

Equating 'P' for both left & right

$= \dfrac{P_1}{(V')^{\gamma}} = \dfrac{P_2}{(V_0 - V')^{\gamma}}$ or $\dfrac{V_0 - V'}{V'} = \left(\dfrac{P_2}{P_1}\right)^{1/\gamma}$

$\Rightarrow \dfrac{V_0}{V'} - 1 = \dfrac{P_2^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow \dfrac{V_0}{V'} = \dfrac{P_2^{1/\gamma} + P_1^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow V' = \dfrac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ For left …….(3)

Similarly $V_0 - V' = \dfrac{V_0 P_2^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ For right ……(4)

(b) Since the whole process takes place in adiabatic surroundings. The separator is adiabatic. Hence heat given to the gas in the left part = Zero.

(c) From (1) Final pressure $P = \dfrac{P_1 \left(\dfrac{V_0}{2}\right)^{y}}{(V')^{\gamma}}$

Again from (3) $V' = \dfrac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ or $P = \dfrac{P_1 \dfrac{(V_0)^{\gamma}}{2^{\gamma}}}{\left(\dfrac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}\right)^{\gamma}} = \dfrac{P_1 (V_0)^{\gamma}}{2^{\gamma}} \times \dfrac{\left(P_1^{1/\gamma} + P_2^{1/\gamma}\right)^{\gamma}}{(V_0)^{\gamma} P_1} = \left(\dfrac{P_1^{1/\gamma} + P_2^{1/\gamma}}{2}\right)^{\gamma}$

31. $A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$, $M = 0.03\ g = 0.03 \times 10^{-3}\ kg$,

$P = 1\ atm = 10^5\ pascal$, $L = 40\ cm = 0.4\ m$.

$L_1 = 80\ cm = 0.8\ m$, $P = 0.355\ atm$

The process is adiabatic

$P(V)^{\gamma} = P(V')^{\gamma} = \Rightarrow 1 \times (AL)^{\gamma} = 0.355 \times (A2L)^{\gamma} \Rightarrow 1\ \text{\textit{1 = 0.355}}\ 2^{\gamma} \Rightarrow \dfrac{1}{0.355} = 2^{\gamma}$

$= \gamma \log 2 = \log\left(\dfrac{1}{0.355}\right) = 1.4941$

$V = \sqrt{\dfrac{\gamma P}{f}} = \sqrt{\dfrac{1.4941 \times 10^5}{m/v}} = \sqrt{\dfrac{1.4941 \times 10^5}{\left(\dfrac{0.03 \times 10^{-3}}{10^{-4} \times 1 \times 0.4}\right)}} = \sqrt{\dfrac{1.441 \times 10^5 \times 4 \times 10^{-5}}{3 \times 10^{-5}}} = 446.33 \approx 447\ m/s$
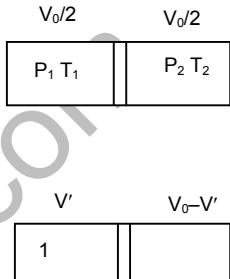
32. V = 1280 m/s,   T = 0°C, foH$_2$ = 0.089 kg/m^3,   rR = 8.3 J/mol-k,

At STP, P = 10^5 Pa, We know

$$V_{sound} = \sqrt{\frac{\gamma P}{fo}} \Rightarrow 1280 = \sqrt{\frac{\gamma \times 10^5}{0.089}} \Rightarrow (1280)^2 = \frac{\gamma \times 10^5}{0.089} \Rightarrow \gamma = \frac{0.089 \times (1280)^2}{10^5} \approx 1.458$$

Again,

$$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.458 - 1} = 18.1 \text{ J/mol-k}$$

Again, $\frac{C_P}{C_V} = \gamma$ or C$_P$ = γC$_V$ = 1.458 × 18.1 = 26.3 J/mol-k

33. μ = 4g = 4 × 10^{-3} kg,   V = 22400 cm^3 = 22400 × 10^{-6} m^3

C$_P$ = 5 cal/mol-ki = 5 × 4.2 J/mol-k = 21 J/mol-k

$$C_P = \frac{\gamma R}{\gamma - 1} = \frac{\gamma \times 8.3}{\gamma - 1}$$

$$\Rightarrow 21(\gamma - 1) = \gamma(8.3) \Rightarrow 21\gamma - 21 = 8.3\gamma \Rightarrow \gamma = \frac{21}{12.7}$$

Since the condition is STP, P = 1 atm = 10^5 pa

$$V = \sqrt{\frac{\gamma f}{f}} = \sqrt{\frac{\frac{21}{12.7} \times 10^5}{\frac{4 \times 10^{-3}}{22400 \times 10^{-6}}}} = \sqrt{\frac{21 \times 10^5 \times 22400 \times 10^{-6}}{12.7 \times 4 \times 10^{-3}}} = 962.28 \text{ m/s}$$

34. Given fo = 1.7 ×10^{-3} g/cm^3 = 1.7 kg/m^3,   P = 1.5 × 10^5 Pa,   R = 8.3 J/mol-k,

f = 3.0 KHz.

Node separation in a Kundt'' tube = $\frac{\lambda}{2}$ = 6 cm, $\Rightarrow \lambda$ = 12 cm = 12 × 10^{-3} m

So, V = $f\lambda$ = 3 × 10^3 × 12 × 10^{-2} = 360 m/s

We know, Speed of sound = $\sqrt{\frac{\gamma P}{fo}} \Rightarrow (360)^2 = \frac{\gamma \times 1.5 \times 10^5}{1.7} \Rightarrow \gamma = \frac{(360)^2 \times 1.7}{1.5 \times 10^5} = 1.4688$

But C$_V$ = $\frac{R}{\gamma - 1} = \frac{8.3}{1.488 - 1}$ = 17.72 J/mol-k

Again $\frac{C_P}{C_V}$ = γ So, C$_P$ = γC$_V$ = 17.72 × 1.468 = 26.01 ≈ 26 J/mol-k

35. f = 5 × 10^3 Hz,   T = 300 Hz, $\frac{\lambda}{2}$ = 3.3 cm $\Rightarrow \lambda$ = 6.6 × 10^{-2} m

V = $f\lambda$ = 5 × 10^3 × 6.6 × 10^{-2} = (66 × 5) m/s

V = $\frac{\lambda P}{f}$ [Pv = nRT $\Rightarrow$ P = $\frac{m}{mV}$ ×Rt $\Rightarrow$ PM = foRT $\Rightarrow \frac{P}{fo} = \frac{RT}{m}$]

$= \sqrt{\frac{\gamma RT}{m}} (66 \times 5) = \sqrt{\frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}}} \Rightarrow (66 \times 5)^2 = \frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}} \Rightarrow \gamma = \frac{(66 \times 5)^2 \times 32 \times 10^{-3}}{8.3 \times 300} = 1.3995$

C$_v$= $\frac{R}{\gamma - 1} = \frac{8.3}{0.3995}$ = 20.7 J/mol-k,

C$_P$ = C$_V$ + R = 20.77 + 8.3 = 29.07 J/mol-k.

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